

Misspecification tests for Periodic Long Memory GARCH models ^(*)

Test di errata specificazione per modelli GARCH periodici a memoria lunga

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Riassunto: In questo lavoro viene studiata la possibilità di applicare i test del rapporto di massima verosimiglianza e dei moltiplicatori di Lagrange come test di errata specificazione di un modello GARCH a memoria lunga periodica. Il corretto funzionamento dei test viene verificato mediante simulazioni Monte Carlo in riferimento alla classe dei modelli GARMA-GARCH.

Keywords: Long Memory, GARMA-GARCH, misspecification tests

1. Introduction

Findings of long memory behavior in the conditional variance are well known in the literature. To face the observed persistence several models were proposed in the econometric literature, the most known are the FIGARCH and the FIEGARCH models (Baillie *et al.* (1996), Andersen and Bollerslev (1997)) and the Long Memory Stochastic Volatility model (Breidt *et al.* (1994)).

More recently, Bordignon, Caporin and Lisi (2005a,b) introduced new models, namely the Period Long Memory GARCH (PLM-GARCH) and the GARMA-GARCH, that generalise the FIGARCH and FIEGARCH models allowing for long memory also of periodic type. The main innovation of these models is the introduction of a generalised long-memory filter which can be applied to most of the GARCH-type structures available in literature. Interesting computational advantages can be obtained considering the logarithmic specification of both models.

In this paper the relation between the PLM and GARMA-GARCH models is taken in account to show that a nesting relation exists between the two. This relation allows appropriate testing schemes for the presence of generalised long memory patterns in high frequency data.

We show, by Monte Carlo simulations, that Likelihood Ratio (LR) and Lagrange Multiplier (LM) tests can be used to detect generalised long memory behaviours in the conditional variance and to verify possible misspecifications. Analytical expressions of the Gradient and of the Hessian of the GARMA-Log-GARCH model are also computed. Among others thing, these quantities allow a faster and more precise numerical optimization in the quasi maximum likelihood context.

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2. Periodic long memory filter and GARMA – GARCH models

According to Woodard *et al.* (1988), a k-factor Generalized ARMA (GARMA) model that allows for long memory behavior associated with k frequencies in $[0, \pi]$ is defined by

$$\phi(L) \prod_{i=1}^k (1 - 2\eta_i L + L^2)^{d_i} (y_t - \mu) = \Theta(L)\varepsilon_t \quad (1)$$

where k is an integer, ε_t is a white noise with variance σ_ε^2 , μ is the mean of the process, η_i ($i = 1, \dots, k$) specifies the frequency at which the long-memory behaviour occurs and d_i is the long-memory parameter indicating how slowly the autocorrelations damp.

The characteristics of model (1) is the presence of the generalized long-memory filter $P(L) = \prod_{i=1}^k (1 - 2\eta_i L + L^2)^{d_i}$ that models the long memory periodic behavior. When we think of the frequencies associated to η_i as the driving frequencies of a cyclical pattern of length S we have $\eta_i = \cos\left(\frac{2\pi(i-1)}{S}\right)$ and $k = [S/2] + 1$, where $[\cdot]$ stands for the integer part.

It should be noted that also standard seasonal filters, such as $(1 - L^4)^d$, used for quarterly data, can be recast into the $P(L)$ form evidencing the contribution of each associated periodic frequency. Infact

$$(1 - L^4)^d = (1 - \eta_1 L + L^2)^d (1 - \eta_2 L + L^2)^d (1 - \eta_3 L + L^2)^d$$

with $\eta_1 = 1, \eta_2 = 0, \eta_3 = -1$.

Bordignon, Caporin and Lisi (2005b) proposed to include the generalized long-memory filter $P(L)$ into a GARCH structure to describe the periodic pattern observed in the intradaily financial time series. The class of models they proposed is called GARMA-GARCH. Actually, this extension is not always feasible due to the constraints needed for conditional variance positivity. For this reason, Bordignon, Caporin and Lisi (2005b) suggested to use the logarithmic specification which is easier to estimate. The corresponding GARMA-Log-GARCH model is given by

$$y_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t z_t \quad \varepsilon_t | I_{t-1} \sim D(0, \sigma_t^2)$$

where μ_t is the conditional mean of y_t , z_t is an i.i.d. random variable with zero mean while σ_t^2 is, as usual, the conditional variance whose dynamics is

$$\begin{aligned} \ln(\sigma_t^2) = & \omega + \beta(L) \ln(\sigma_t^2) + \\ & + \left[1 - \beta(L) - \left[\prod_{i=1}^k (1 - 2\eta_i L + L^2)^{d_i} \right] (1 - \phi(L)) \right] [\ln(\varepsilon_t^2) - k(\theta)] \end{aligned} \quad (2)$$

where $k(\theta) = E[\ln(z_t^2)]$, equal to -1.27 in the gaussian case, and $\beta(L)$ and $\phi(L)$ are suitable polynomials in L . In the GARMA-GARCH framework, thus, each periodic frequency is modelled by means of a specific long-memory parameter d_i .

The Periodic Long Memory Log-GARCH model (PLM-L-GARCH) proposed by Bordignon, Caporin and Lisi (2005a) is a particular case of the GARMA-Log-GARCH which occurs when $d_i = d \ \forall i$. In this case expression (2) can be written as

$$\ln(\sigma_t^2) = \omega + \beta(L) \ln(\sigma_t^2) + \left[1 - \beta(L) - (1 - L^S)^d (1 - \phi(L)) \right] [\ln(\varepsilon_t^2) - k(\theta)] \quad (3)$$

which can be derived from the previous model under a set of parameter restrictions. It is clear that model (2) is more flexible and can face more complex behaviors than model (3), however the latter is more parsimonious and give an easier description of the periodic component. It is useful, thus, to have suitable tests for establishing when use the GARMA-GARCH or the PLM-GARCH. Similarly, we can be interested in testing the opportunity of fitting a long-memory periodic model rather than a short-memory periodic one.

Since the GARMA-GARCH class encompasses PLM-GARCH as well as other short-memory GARCH specifications, it is possible to exploit this nesting relationship to apply the standard LR or LM tests as a misspecification test. For example, to test the a PLM-GARCH form *versus* a possible GARMA-GARCH specification imply verifying the hypothesis $H_0 : d_i = d \ (i = 1, \dots, k)$, while testing PLM-GARCH or GARMA-GARCH forms *versus* a short-memory GARCH with coefficients at periodic lags one can test the hypothesis $H_0 : d_i = 0 \ (i = 1, \dots, k)$ which induces a GARCH model were $\alpha(L) = \phi(L) - \beta(L)$. As usual, LR test can be applied only to nested models and LM test requires to compute derivatives of the likelihood. The analytical gradient and Hessian matrix were calculated by the authors but non included in the present short-paper for brevity.

3. Monte Carlo simulations

In order to proof the proper working of our testing approach we compare nominal and real levels through Monte Carlo simulations. In our experiments we consider $M = 1000$ simulation trials for series of length $n = 500, 1000$ and 2000 . All models have no mean component ($\mu_t = 0$), a periodic component of period $S = 7 \ (k = 4)$ and a constant on the conditional variance equal to $\omega = -0.5$ or to $\omega = -0.1$. The models involved in the simulations are generated from three different data generating processes:

- M1: a GARMA-Log-GARCH with $\beta(L) = 0, \phi(L) = 0$, with parameters d_1, d_2, d_3 and d_4 not all equal and covering cases with both high and low memory coefficients;
- M2: a PLM-Log-GARCH with $\beta(L) = 0, \phi(L) = 0$, and parameters $d_1 = d_2 = d_3 = d_4 = d$ set to 0.1, 0.2, 0.4 or 0.7;
- M3: a short-memory GARCH with coefficients at lags 1 and S both for $\alpha(L)$ and $\beta(L)$ and inducing a persistent GARCH behavior.

Within these cases we test the nesting hypothesis between the three DGPs, in details, PLM versus GARMA, GARCH versus PLM and GARCH versus GARMA.

In this paper we report very preliminary results on two cases for the test between PLM and GARMA specifications, namely $H_0 : d_i = d \ (i = 1, \dots, k)$. Table 1 evidences that both the LM and LR tests clearly suggest the rejection of the null hypothesis when the true DGP is of the GARMA-type, while the rejection frequencies are close to the confidence levels when the true DGP is of the PLM-type. These are very preliminary results but they seem to be encouraging suggesting that the proposed approach could be used in long-memory GARCH framework for testing for the presence of periodic long-memory in two specific forms.

True parameters	n	LM test			LR test		
		1%	5%	10%	1%	5%	10%
d1=0.10 d2=0.20 d3=0.25 d4=0.25	1000	1	1	1	1	1	1
d1=0.10 d2=0.10 d3=0.10 d4=0.10	1000	0.016	0.059	0.118	0.010	0.053	0.096

Table 1: *Nominal and real levels for LR and LM test. The real levels are based on $M = 1000$ simulation trials.*

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